

Vortex structure in mesoscopic superconductors

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The nonlinear Ginzburg-Landau equations are solved numerically in order to investigate the vortex structure in thin superconducting disks of arbitrary shape. Depending on the size of the system and the strength of the applied magnetic field giant vortex, multi-vortex and a combination of both of them are found. The saddle points in the energy landscape are identified from which we obtain the energy barriers for flux penetration and expulsion.

keywords: vortex matter, mesoscopic, flux penetration, flux quantization

I. INTRODUCTION

Recent advances in nanoscience have demonstrated that fundamentally new physical phenomena are found when the size of the sample shrinks and becomes comparable to the length scale of the investigated phenomenon.

Superconductivity is a macroscopic quantum phenomenon and therefore it is interesting to see how this quantum state is influenced if the sample is reduced in size. For rings and hollow cylinders this leads to the well-known Little-Parks effect [1] which results in a periodic variation of the critical temperature as function of the applied magnetic field, the period being determined by the magnetic flux value within the tube.

Bulk superconductors are divided up into type-I ($\kappa < 1/\sqrt{2}$) and type-II ($\kappa > 1/\sqrt{2}$) superconductors, the distinction between them is completely determined by the Ginzburg-Landau parameter $\kappa = \lambda/\xi$, where λ is the magnetic field penetration depth and ξ the coherence length, which is a material parameter. The difference is clearly seen in the magnetic response of the system, where type-I has a complete Meissner effect while type-II superconductors can have a state with partial expulsion of the field in which flux lines penetrate the superconductor. The latter is a superconducting state with disconnected circular areas of normal state. For mesoscopic superconductors we find that type-I superconductors can behave like type-II or even show some mixed behaviour depending on the size of the system and therefore κ is no longer the only determining parameter characterizing the vortex state of the system.

The vortex structure in superconductors will be described in the framework of the Ginzburg-Landau (GL) theory, which consists of two coupled nonlinear differential equations:

$$\left(-i\vec{\nabla}_{2D} - \vec{A}\right)^2 \Psi = \Psi \left(1 - |\Psi|^2\right), \quad (1)$$

$$-\Delta_{3D} \vec{A} = \frac{d}{\kappa^2} \delta(z) \vec{j}_{2D}, \quad (2)$$

where

$$\vec{j}_{2D} = \frac{1}{2i} \left(\Psi^* \vec{\nabla}_{2D} \Psi - \Psi \vec{\nabla}_{2D} \Psi^* \right) - |\Psi|^2 \vec{A}, \quad (3)$$

is the density of superconducting current and d the thickness of the sample. The superconducting wavefunction satisfies the boundary conditions $\left(-i\vec{\nabla}_{2D} - \vec{A}\right) \Psi \Big|_n = 0$ normal to the sample surface and $\vec{A} = \frac{1}{2} H_0 \rho \vec{e}_\phi$ far away from the superconductor. The distances are measured in units of the coherence length $\xi = \hbar/\sqrt{-2m^*\alpha}$, the order parameter in $\Psi_0 = \sqrt{-\alpha/\beta}$ and the vector potential in $c\hbar/2e\xi$. $\kappa = \lambda/\xi$ is the Ginzburg-Landau parameter, and $\lambda = c\sqrt{m/\pi}/4e\Psi_0$ is the penetration length. The magnetic field is measured in $H_{c2} = c\hbar/2e\xi^2 = \kappa\sqrt{2}H_c$, where $H_c = \sqrt{4\pi\alpha^2/\beta}$ is the critical field. Temperature is included in ξ , λ , H_{c2} , through their temperature dependencies: $\xi(T) = \xi(0)/\sqrt{|1 - T/T_{c0}|}$, $\lambda(T) = \lambda(0)/\sqrt{|1 - T/T_{c0}|}$, $H_{c2}(T) = H_{c2}(0)|1 - T/T_{c0}|$, where T_{c0} is the critical temperature at zero magnetic field.

Different approaches have been used to solve the GL-equations. In the lowest Landau level approximation [2] a linear combination of solutions of the linearized first GL-equation is used and the internal (taken homogeneous) magnetic field and the expansion coefficients are determined by minimizing the free energy. An extension of this approach beyond the lowest Landau level was given in Ref. [3]. It was shown recently [4] that in order to account for demagnetization effects in the lowest Landau level approximation it is necessary to introduce an effective κ which

depends on the size of the superconductor. Trial functions [5] with several variational parameters were used to study the vortex configuration in mesoscopic cylinders with suppressed surface superconductivity. Here, we will solve the two GL equations numerically using a finite difference technique. Details of this approach can be found in Ref. [6].

II. GIANT VORTEX TO MULTI-VORTEX TRANSITION

First we consider a circular disk with thickness d . Because of the circular symmetry we assume that the order parameter is cylindrical symmetric: $\Psi(x, y) = F(\rho)e^{-L\phi}$ where L is the vorticity. In doing so we restrict our set of solutions to a subset in which the modulus of the local order parameter is axially symmetric.

Because of the non-linear term in the GL-equation the superconducting state is in general non-axial symmetric even if the sample is circular symmetric. A well-known example is the Abrikosov lattice which has triangular symmetry. We found that for our circular disks such non-axial symmetric states are found when the radius of the disk is sufficiently large and when the magnetic field is not too large. For small disks the boundary condition dominates which imposes its symmetry on the order parameter. For large magnetic fields the inner part of the disk becomes normal and the superconducting state survives only near the sample surface where the shape of it will again determine the symmetry of the order parameter. The free energy of the superconducting disk is shown in Fig. 1(a) as function of the magnetic field for different values of the vorticity. The full curves are for the giant vortex state and the dashed curves for the multi-vortex states. The transition point between them is given by the open dot. Figs. 1(c,d,e) shows a contour plot of the modulus of the superconducting density for $L = 3$ at $H_0/H_{c2} = 0.62, 0.72$ and 0.82 , respectively. Notice that with increasing magnetic field the size of the vortices grows and they move closer to each other. At the transition field the single vortices coalesce into one giant vortex. This is a continuous transition and therefore of second order [7].

The magnetization is shown in Fig. 1(b) where the vertical lines indicate the ground state transitions. Notice that for small magnetic fields we have a linear $M - H$ relation which is typical for an ideal diamagnet. With increasing magnetic field there is a *continuous* penetration of the magnetic field at the edge of the sample which leads to a smaller than linear increase of M with H . This effect is enhanced by demagnetization effects which leads to an enhanced magnetic field at the edge of the sample (see inset of Fig. 1(a)). When we further increase the magnetic field the energy of the superconductor increases up to the point where it is energetically more favorable to transit to the $L = 1$ state. As a consequence $M(H)$ exhibits a zig-zag behavior which was measured recently by A. Geim *et al* [8] and explained in Ref. [9]. This behavior is due to the fact that with increasing magnetic field more field can penetrate into the sample in a *discontinuous* way in which the vorticity of the order parameter increases with one unit. Approximately (because of boundary effects), one unit of flux enters the superconductor. Therefore, by counting the number of jumps in the $M(H)$ curve it is possible to count the vorticity of the sample and therefore the number of vortices inside the disk.

Notice that for a given magnetic field different states are possible. Experimentally, it has been possible to drive the system into the different metastable regions and to map out the complete magnetization-magnetic field curves as shown in Fig. 1(b) by slowly ramping the field up and down. These metastable states are responsible for hysteretic behavior which was observed in the magnetic field response of superconducting disks. A detailed comparison between experimental results and our theoretical calculations can be found in Ref. [10]. It was shown recently [11] that in certain samples the superconducting state can be forced very far into the metastable region such that fractional flux can penetrate and in some cases even negative flux penetration is observed for the transition $L \rightarrow L + 1$.

III. GEOMETRY DEPENDENCE

One may wonder how important the exact shape of the sample is for the symmetry of the order parameter. To investigate this, we took as an example a thin flat triangular shaped superconducting sample. For the free energy we found a similar behavior as depicted in Fig. 1(a) but with the following major differences: 1) the critical magnetic field H_{c3} at which the normal state is reached is substantially larger, i.e. for a triangle with the same surface area as the disk of Fig. 1(a) we obtained $H_{c3}/H_{c2} = 2.5$. This is a consequence of the enhanced surface conductivity in wedge shaped samples [12,13]; 2) The multi-vortex state (see Fig. 2(a)) is substantially stabilized, i.e. the magnetic field range over which it is stable is strongly enhanced. Furthermore, for certain values of the vorticity only the multi-vortex state is stable, and the giant vortex state does not occur. The giant vortex state is now no longer circular symmetric, but it is characterized by the fact that the order parameter has only one zero. 3) A new vortex structure appears which is a combination of a giant vortex in the center of the triangle with single vortices around it. This state occurs e.g. for $L = 5$; 4) From the contour plot of the superconducting current flow (see Fig. 2(b)) we see that the screening

currents along the edge flow clockwise while the currents around the vortices flow counterclockwise. Notice that near the edges there are clockwise spiralling local vortex-like currents which are not connected to the position of a vortex (see the contour plot of Fig. 2(c) for the phase of the order parameter) but are rather back-flow currents which are well-known in hydrodynamics.

Recently, Chibotaru and co-workers [14] studied the linearized GL-equation for triangles and squares and found that the symmetry of the boundary imposes a similar symmetry on the superconducting state. For example, in case of the triangle and for $L = 2$ this occurs by having three vortices sitting in a triangular arrangement and one anti-vortex in the center of this triangle. From the outset we should point out that the linearized GL-equation is only strictly valid at the superconducting/normal boundary where the modulus of the order parameter approaches zero. It was found recently [15] that once one moves away from this phase line the anti-vortices are quickly annihilated. Then a state is found in which a large central area of the triangle is normal and only superconductivity survives in the corners of the triangle [16]. The anti-vortices and the vortices are found in the central area of the triangle where $|\Psi|^2$ is extremely small. Therefore, the experimental relevance of this new state is questionable.

IV. DIFFERENT ARRANGEMENTS OF MULTI-VORTICES

When the superconducting state is in the multi-vortex state, it is possible that different vortex configurations are possible. This is illustrated in Fig. 3 where for a disk of radius $R/\xi = 6$ the energy of the $L = 6$ and $L = 7$ vortex states are plotted as function of the magnetic field. In this case a multi-vortex configuration is possible with (dashed curves) or without (solid curves) a vortex in the center of the disk. The corresponding contourplots of the superconducting density in the disk is shown in the insets of Fig. 3. The states are indicated by $(n; L)$ where n refers to the number of vortices in the center and L is the vorticity of the superconducting state. Notice that for $L = 6$ the $(1;6)$ state has a larger energy than $(0;6)$ when $H/H_{c2} > 0.45$ which then becomes the ground state. This change of configuration at $H/H_{c2} \sim .45$ is a first order transition because it involves a change of symmetry of the superconducting state. At $H/H_{c2} \sim 0.6$ the ground state transits to a higher vorticity state with configuration $(1;7)$. This transition is also a first order transition in which the magnetization is discontinuous. Such transitions have been found experimentally [17].

Notice that the appearance of the different multi-vortex configurations is very similar to the classical system of interacting (repulsion) particles which are confined into a potential. It was found that the particles are situated in ring-like configurations [18], very similar to the vortex configurations found in Fig. 3. Also in this case different meta-stable states consisting of different particle configurations are found.

V. FLUX PENETRATION AND EXPULSION

In the above discussion we considered only the local minima in the energy functional which determine the ground and metastable states. The transition between states with different vorticity does not always follow the ground state. The reason is that there are barriers for flux penetration and expulsion. The most well-known is the Bean-Livingston model [19] for the *surface barrier* which is a result of the competition between the vortex attraction to the sample walls by its mirror image and its repulsion by screening currents. For nonelliptical samples a *geometrical barrier* appears because of Meissner currents flowing on the top and bottom surface [20]. Additionally, *vortex pinning* by defects can play an important role in the delay of vortex expulsion or promotion of vortex penetration. These models describe the vortex formation far from the sample boundary.

As an example, we will consider here mesoscopic disks in which boundary effects are predominant and consequently previous approaches are not applicable. We consider a defect-free thin superconducting disk with a perfect circular boundary such that $Rd/\lambda^2 \ll 1$. When calculating the free energy we not only search for the local minima but also for the saddle points which are the lowest energy barriers between two such minima. The details of the calculation can be found in Refs. [3,21].

The results are shown in Fig. 4 where the energy of the saddle point is given by the dotted curve. In contrast to known surface and geometrical barrier models, we find that in a wide range of magnetic fields below the penetration field, the saddle point state for flux penetration into a disk does not correspond to a vortex located near the sample boundary, but to a region of suppressed superconductivity (see Figs. 4(a-d)) at the disk edge with no winding of the current (Fig. 4(c)), and which is a *nucleus* for the following vortex creation. The height of this *nucleation barrier* is shown in the inset of Fig. 4 and determines the time of flux penetration.

VI. CONCLUSIONS

The vortex state of a mesoscopic superconductor is strongly determined by its size and to a lesser extend by the material parameters the superconductor is made of. For example, by increasing the radius (starting from $R \ll \xi$) of a superconducting disk [8,9] it is possible to obtain a magnetic response which, as function of the magnetic field, is continuous, type-I, a type-I with multiple steps and type-II. The exact geometry of the superconductor has also a strong influence on its vortex state. This vortex state can be brought into a metastable region owing to the presence of barriers for flux motion which leads to unexpected effects like fractional flux and even negative flux entry. In mesoscopic superconductors the flux in general is not quantized [11]. This is even more so in small superconducting rings [22]. The lowest energy barrier between two flux states was identified and corresponds to a saddle point of the energy functional.

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FIG. 1. (a) The free energy and (b) the magnetization of the multi-vortex states (dashed curves) and the giant vortex states (solid curves) as a function of the applied magnetic field for a disk with radius $R = 4.0\xi$ and thickness $d = 0.1\xi$ ($\kappa = 0.28$). The inset in (a) shows the radial magnetic field distribution through the central plane of the disk for the $L = 3$ state at $H_0/H_{c2} = 0.82$ (giant vortex state). The open circles indicate the transition from multi-vortex to giant-vortex states. (c-e) Contour plots of the Cooper-pair density for the $L = 3$ state at $H_0/H_{c2} = 0.62, 0.72$ (multi-vortex states) and 0.82 (giant vortex state). High (low) Cooper-pair density is given by dark (light) regions.

FIG. 2. (a) The Cooper-pair density, (b) the supercurrent and (c) the phase of the order parameter for a triangle with width $W = 10.774\xi$ and thickness $d = 0.1\xi$ ($\kappa = 0.28$). The applied field is $H_0 = 0.495H_{c2}$ and $L = 2$.

FIG. 3. The free energy F as a function of the applied magnetic field H_0 of the $(0;6)$ and $(0;7)$ state (solid curves), and the $(1;6)$ and $(1;7)$ state (dashed curves) for a superconducting disk with radius $R = 6.0\xi$. The insets show the Cooper-pair density of the four different states at $H_0/H_{c2} = 0.6$.

FIG. 4. The free energy as a function of the applied magnetic field H_0 for a circular disk of radius $R/\xi = 4$. The solid curves correspond to the giant vortex state, the dashed curves are the multi-vortex states and the dotted curves are the energy of the saddle point. The inset shows the lowest energy barrier for the transition $L \rightarrow L + 1$. (a-d) show contour plots of the superconducting density for the saddle point corresponding to the $L = 4 \rightarrow 5$ transition at the magnetic fields $H_0/H_{c2} = 0.81, 0.885, 0.96$ (the barrier maximum), 1.035 , respectively.







